Everything is fractal

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Abstract

In this work the fractal geometry and the box dimension are introduced and their role in the description of irregular and broken shapes is shown. Examples of fractals coming from the world of cultural heritage are presented, and a typical fractal measurement on an art work is discussed.

Introduction

The fractal geometry has been invented (or discovered) by Benoit B. Mandelbrot in 1967 and it is now widely recognized as the true geometry of nature: "Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line" [1]. Everything around us is irregular, chaotic, fragmented, complex and rough; only the fractal geometry appears to be a practical way to measure and characterize the irregularity of natural shape of rocks, river basin, trees, buildings, and cities.

What are fractals

Fractals are self-similar complex objects, i.e. structures composed of many parts that when magnified have (exactly or statistically) the same shape of the whole. This is the case of well known mathematical sets (e.g. Koch snowflake, Sierpinski gasket, Menger sponge) as well as many natural things coming from geology, anatomy, botany and so on. The shape of a coast, the basin of a river the surface of a rock, the vascular tree, the lung branching, and the roots of a plant are all examples of natural fractals. Their self-similarity is well recognized observing that an enlarged picture of the finest details resembles more or less the object itself.

Scientist and artists have observed this scale invariance since long time before the discovering of fractals and fractal language. In 1500 Leonardo da Vinci stated in the Leicester code: "The waters of great floods make the same revolutions in their ways than the little waters do"; in 1300 Cennino Cennini, an Italian painter, wrote in his book of the art: "If you want to picture a mountain take a big wrinkled stone and copy it".

Fractal is also the maths of chaos. Deterministic chaos, i.e. the apparent erratic behaviour of nonlinear dynamical systems, was discovered in 1963 by the meteorologist Edward N. Lorenz, who coined the term "butterfly effect" to express the sensitive dependence on initial conditions. This phenomenon is considered today the fingerprint of chaos and it is a consequence on the fractal properties of the strange attractor in the phase space portrait of a chaotic system.

Therefore, both static and dynamic properties of nature appears to be self-similar, and this extends the domain of fractal geometry to sciences traditionally excluded from mathematical analysis, such as finance, sociology, anthropology, medicine.

The fractal dimension

A very useful parameter widely employed for the numerical characterization of a fractal is the fractal dimension. Its rigorous mathematical definition is derived from the Hausdorff-Besicovitch dimension [2]; for our purpose it is sufficient to say that the fractal dimension can be considered as a measure of the degree of irregularity of a complex shape, or the extent of the space filling capacity. Surprisingly this dimension can be a non integer number, in contrast with the dimensions of Euclidean geometrical forms: zero for a point, one for a line, two for a surface, and three for a solid. Usually fractals have fractional dimensions: for example an object whose dimension is comprised between one and two is more complex than a regular curve, bur less space filling than the whole plane.

Two kinds of fractal dimension are useful in fractal analysis: the similarity dimension and the box-counting dimension. The former is readily applied to exact self-similar mathematical object, the latter is suitable for statistical fractals, just like those observed in nature. In practice it is calculated by laying a square lattice on the digitalized image of the real object and calculating the number of boxes containing almost one element of the structure. The fractal dimension is obtained as the slope of the log-log plot of the number of occupied box as a function of the box side length.

In fig. 1 the fractal analysis of a painting of Jackson Pollock (entitled “Number 18”) from Guggenheim, recently exposed in Rome at Palazzo delle Esposizioni, is reported. The exponent 1.75 of the power law is the mean box dimension of the picture [3]. A more detailed analysis (not shown here) gives two different fractal dimensions, 1.49 at low box lengths and 1.87 at higher lengths, indicating that the drawing is nearly Euclidean at low resolution due to the large colour patches.

Figure 1. **left:** the original colour picture of Pollock; **center:** the same image converted in binary black and white dots; **right:** the log-log plot of the binary image showing a fractal scaling and the value of the box dimension.

Fractals in cultural heritage

A lot of art works exhibit fractal self-similarity. Typical examples include Magritte leaf shaped trees, Hokusai sea waves, Escher tessellation prints, and Pollock dripping paintings (see above). Some architectures are fractal as well: just to mention few, consider the Eiffel tower in Paris, the Indonesian Prambanan temple, and the Taj Mahal love memorial in Agra. In a more generalized definition [4], every object showing new and interesting details at every scale of magnification can be considered a fractal. In this way the list of fractal architectures can be extended to include Indian temples, gothic cathedrals, downtown of old cities (e.g. the UNESCO world heritage site of Matera), ancient Maya settlements, and even the bodily structure of all living form. In this sense we can say that everything is fractal.

In addition a number of experimental data demonstrate the existence of fractal structure in archaeological patterns as well as in natural geological porous media. This information can be used to improve our knowledge of the soil and the building material in a perspective of study on degradation and restoration. Also the aging of polymers can be studied through fractal models.

Conclusions

In recent years fractal analysis has been revealed a very useful tool for quantitative description of irregular and fragmented objects, coming from every field of science. Nevertheless the application of such a tool in the humanities is still at the beginning. We hope that in the future it will be widely employed to shed a new light on the knowledge of historical artefacts.

References

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